3D MRI-based retinal shape determination – Appendix A

Supplementary methods

Luc van Vught,[1,2] Denis P. Shamonin,[3] Gregorius P.M. Luyten,[1] Berend C. Stoel[3] and Jan-Willem M. Beenakker[1,2]

1) *Department of Ophthalmology, Leiden University Medical Center, Leiden, the Netherlands*
2) *Department of Radiology, C.J. Gorter Center for High Field MRI, Leiden University Medical Center, Leiden, the Netherlands*
3) *Department of Radiology, Division of Image Processing (LKEB), Leiden University Medical Center, Leiden, the Netherlands*

1. MATHEMATICAL DESCRIPTION OF THE EYE ROTATIONS

As the gazing direction during the MRI-examination will differ between subjects, there will be small rotational differences between the segmented eyes of these subjects. To allow for comparisons between subjects, all segmentations were aligned towards the same coordinate system. To this end, the segmentations were rotated around the center of the lens to orient the central axis of the eye parallel to the z-axis. Three different rotation-methods, a head-fixed rotation method, an eye-fixed rotation method and an Euler rotation method were evaluated (figure A1). The mathematical description of these rotation-methods is given below.

1.1 HEAD-FIXED ROTATION

The head-fixed rotation method is applied by performing two subsequent rotations, an initial rotation around the feed-head (FH) axis and a second rotation around the left-right (LR) axis. In our implementation, these axes are defined as the coordinate system of the MRI scanner.

Let $\hat{\mathbf{g}}_{\text{init}}$ be the unit vector pointing from the center of the lens to the center of the vitreous body and $\hat{\mathbf{g}}_{\text{final}}$ the unit vector in the desired direction. The angle for the rotation around the FH-axis, θ_{FH}, is defined by $\arccos(\hat{\mathbf{g}}_{\text{proj}} \cdot \hat{\mathbf{g}}_{\text{final}})$, with $\hat{\mathbf{g}}_{\text{proj}}$ the normalized projection of $\hat{\mathbf{g}}_{\text{init}}$ onto the AP-LR-plane. The intermediate gaze direction, $\hat{\mathbf{g}}_{\text{inter}}$ is defined by:

$$\hat{\mathbf{g}}_{\text{inter}} = \boldsymbol{R}_{\text{FH}}(\theta_{FH})\hat{\mathbf{g}}_{\text{init}}, \tag{1.1}$$

with $\boldsymbol{R}_{\text{FH}}$ the rotation matrix for a rotation around the FH-axis.

The angle for the subsequent rotation around LR-axis, θ_{LR}, is calculated similarly: $\arccos(\hat{\mathbf{g}}_{\text{inter,proj}} \cdot \hat{\mathbf{g}}_{\text{final}})$, with $\hat{\mathbf{g}}_{\text{inter,proj}}$ the normalized projection of $\hat{\mathbf{g}}_{\text{proj}}$ onto the AP-FH-plane.

With $\boldsymbol{R}_{\text{LR}}$, the rotation matrix for a rotation around the LR-axis, the full transformation can be defined by:

$$\boldsymbol{R}_{\text{head}} = \boldsymbol{R}_{\text{FH}}(\theta_{FH})\boldsymbol{R}_{\text{LR}}(\theta_{LR}) \tag{1.2}$$

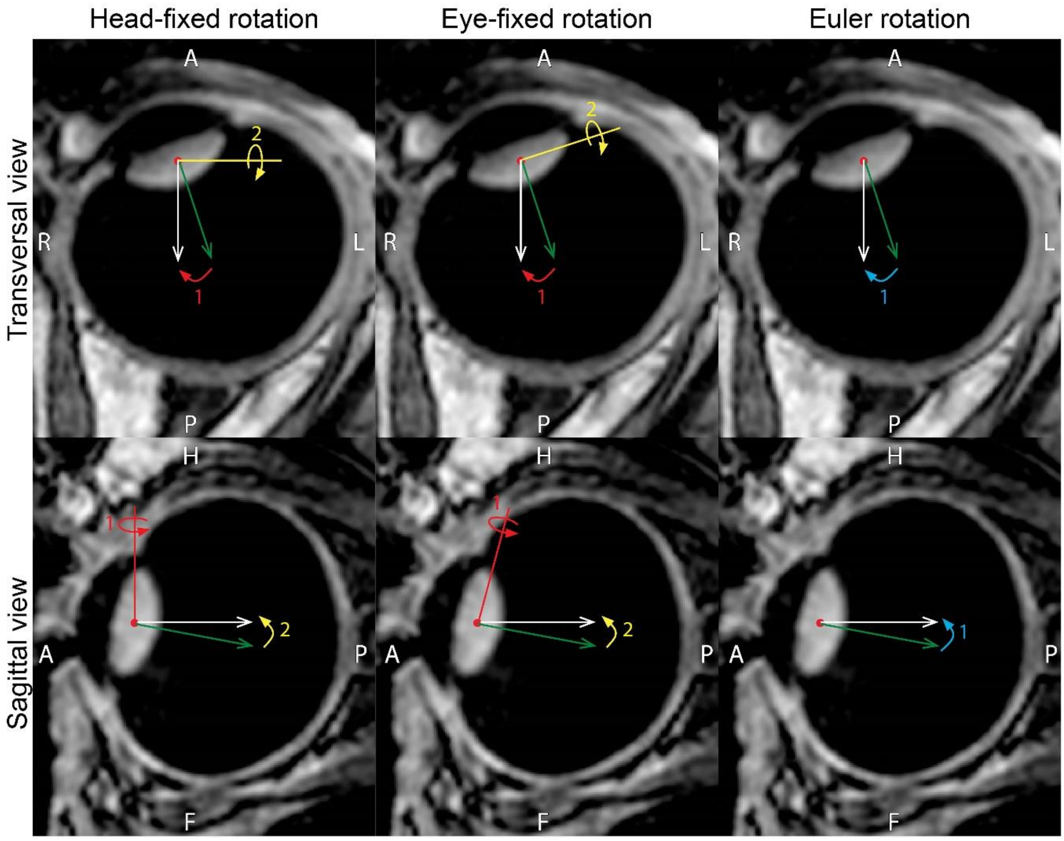

Figure A1: Overview of the three methods used to align the MR-segmentations to the same orientation in transversal and sagittal view. Left: the first (red) and second (yellow) rotation performed using a head-fixed rotation method. Middle: the first (red) and second (yellow) rotation performed using an eye-fixed rotation method. Right: the only (blue) rotation performed using an Euler rotation method.

1.2 EYE-FIXED ROTATION

The second method to align data is similar to the head-fixed method, but the axes of rotation are defined with respect to the eye instead of the head. As a result, the axis of rotation for the second rotation depends on the first rotation. To this end an eye-fixed coordinate system is constructed in which the rotations are calculated in this coordinate system, using a transformation-matrix to express all the vectors in the eye-fixed coordinate system.

Let $\hat{\mathbf{g}}_{\mathrm{init}}$ be the unit vector pointing from the center of the lens to the center of the vitreous body, $\hat{\mathbf{g}}_{\mathrm{final}}$ the unit vector in the desired direction and $\hat{\mathbf{u}}_{FH}$ the unit vector in FH-direction, all in the original, head-fixed, coordinate system. The basis-set for the eye-fixed coordinate system, $\hat{\mathbf{h}}_1, \hat{\mathbf{h}}_2, \hat{\mathbf{h}}_3$ is defined as:

$$\hat{\mathbf{h}}_1 = \hat{\mathbf{g}}_{\text{init}} \tag{1.3}$$

$$\mathbf{h}_{\text{tmp}} = \hat{\mathbf{u}}_{FH} - (\hat{\mathbf{u}}_{FH} \cdot \hat{\mathbf{g}}_{\text{init}})\hat{\mathbf{g}}_{\text{init}} \tag{1.4}$$

$$\hat{\mathbf{h}}_2 = \frac{\mathbf{h}_{\text{tmp}}}{\left|\mathbf{h}_{\text{tmp}}\right|} \tag{1.5}$$

$$\hat{\mathbf{h}}_3 = \hat{\mathbf{h}}_1 \times \hat{\mathbf{h}}_2 \tag{1.6}$$

and with

$$\boldsymbol{T} = \begin{bmatrix} \hat{\mathbf{h}}_1^T \\ \hat{\mathbf{h}}_2^T \\ \hat{\mathbf{h}}_3^T \end{bmatrix}^{-1} \tag{1.7}$$

as the coordinate transformation matrix from the MRI basis-set to the eye basis-set.

Subsequently, the desired gazing direction is expressed in the eye coordinate system, $\hat{\mathbf{g}}'_{\text{final}} = \boldsymbol{T}\hat{\mathbf{g}}_{\text{final}}$, and the rotation matrix $\boldsymbol{R}'_{\text{eye}}$ within this system is calculated in the same manner as with head-fixed coordinate system, using $\hat{\mathbf{h}}_2$ and $\hat{\mathbf{h}}_3$ as the two axes of rotation and $\hat{\mathbf{g}}'_{\text{final}}$ as target. The final rotation matrix in the MRI coordinate system, $\boldsymbol{R}_{\text{eye}}$, can be obtained by:

$$\boldsymbol{R}_{\text{eye}} = \boldsymbol{T}^{-1}\boldsymbol{R}'_{\text{eye}}\boldsymbol{T} \tag{1.8}$$

1.3 EULER ROTATION

Euler's method uses a single rotation to align the segmented eye-data. Let $\hat{\mathbf{g}}_{\text{init}}$ be the unit vector pointing from the center of the lens to the center of the vitreous body and $\hat{\mathbf{g}}_{\text{final}}$ the unit vector in the desired direction. Euler's rotation theorem defines the needed rotation around the vector $\hat{\mathbf{r}}_e$ of angle θ_e as:

$$\hat{\mathbf{r}}_e = \frac{\hat{\mathbf{g}}_{\text{init}} \times \hat{\mathbf{g}}_{\text{final}}}{|\hat{\mathbf{g}}_{\text{init}} \times \hat{\mathbf{g}}_{\text{final}}|} = \begin{bmatrix} r_x \\ r_y \\ r_z \end{bmatrix} \tag{1.10}$$

$$\theta_e = \arccos(\hat{\mathbf{g}}_{\text{init}} \cdot \hat{\mathbf{g}}_{\text{final}}) \tag{1.11}$$

which can be expressed as the matrix $\boldsymbol{R}_{\text{Euler}}$:

$$\hat{\mathbf{g}}_{\text{final}} = \boldsymbol{R}_{\text{Euler}}\hat{\mathbf{g}}_{\text{init}} \tag{1.12}$$

$$\boldsymbol{R}_{\text{Euler}} = \begin{bmatrix} 1 + (1 - \cos\theta_e)(r_x^2 - 1) & -r_z \sin\theta_e + (1 - \cos\theta_e)r_x r_y & r_y \sin\theta_e + (1 - \cos\theta_e)r_x r_z \\ r_z \sin\theta_e + (1 - \cos\theta_e)r_x r_y & 1 + (1 - \cos\theta_e)(r_y^2 - 1) & -r_x \sin\theta_e + (1 - \cos\theta_e)r_y r_z \\ -r_y \sin\theta_e + (1 - \cos\theta_e)r_x r_z & r_x \sin\theta_e + (1 - \cos\theta_e)r_y r_z & 1 + (1 - \cos\theta_e)(r_z^2 - 1) \end{bmatrix} \tag{1.13}$$

2. 3D FITTING ALGORITHM

The fitted 3D ellipsoid was defined as:

$$\frac{x'^2}{R_{x'}} + \frac{y'^2}{R_{y'}} + \frac{z'^2}{R_{z'}} = 1 \tag{2.1}$$

were x', y' and z' are the three principal axes, referred to as the horizontal, vertical and central axis within the main manuscript, and $R_{x'}$, $R_{y'}$ and $R_{z'}$ are the radii of the ellipsoid along these . These radii, together with the center coordinates of the ellipsoid, $C_{x'y'z'}$, and the angulations of its principal axes, α, β and γ provide nine degrees of freedom to the fitting algorithm.

The fitting algorithm consists of three phases, being the generation of an initial guess, the determination of the center coordinates $C_{x'y'z'}$and the determination of the radii $R_{x'}$, $R_{y'}$ and $R_{z'}$ and angulations α, β and γ. In each phase, the optimal parameters were determined by minimizing the error between the ellipsoid and the contour points using SLSQP-minimization.[1] To this end, the fitting algorithm follows the same protocol, sometimes with minor deviations, as listed below:

1. The selected retinal contour points are translated by $-[C_{x'}, C_{y'}, C_{z'}]$, making the center of the ellipsoid the origin of the coordinate system.
2. The retinal contour points are rotated around the center of the ellipsoid by angles α, β and γ.
3. As the ellipsoid is symmetrical in the principal axes coordinate system, all contour points are mapped to the first quadrant, $x' \to |x'|$, $y' \to |y'|$ and $z' \to |z'|$.
4. The shortest distance between each contour point and the ellipsoid defined by equation 2.1 is calculated. As there is no algebraic expression to calculate the distance between a point and a 3D ellipsoid, an iterative minimization is performed using the quasi-Newton BFGS method.[2] This algorithm finds the location on the ellipsoid, defined by the angles θ and ϑ, that has the shortest distance, d, to the contour point:

$$d^2 = (R_{x'} \cos\theta \sin\phi - x')^2 + \left(R_{y'} \sin\theta \cos\phi \ - y'\right)^2 + (R_{z'} \cos\phi - z')^2 \tag{2.2}$$

   The sum of all squared distances is returned to the fitting algorithm as a measure of the goodness of the fit.

In the first fitting phase, an initial estimate of the center of the ellipsoid and its radii is made, which is used as an initial guess for the following steps. As such an initial guess does not need the highest accuracy, step 4 of the fitting protocol was adjusted such that θ and ϕ are not iteratively determined, but defined by $\theta = \tan^{-1}(y'/x')$ and $\phi = \cos^{-1}\left(z'/\sqrt{x' + y' + z'}\right)$. This approximation has a bias towards a more spherical ellipsoid and is therefore not suitable for the final assessment of the retinal shape, but is multiple orders of magnitude faster than the unbiased iterative distance determination and therefore suitable for providing an initial guess.

In the second fitting phase, the center of the ellipsoid is determined. This center is determined separately from the other parameters as the angulations of the ellipsoid are closely related to its radii and center coordinates, especially if only a relatively small portion of the retina is used as an input. In this intermediate phase, the fitting is performed with the angulations fixed to 0 degrees. This phase is further accelerated by using an adapted ellipsoid description with a single parameter for the horizontal and vertical ellipsoid radii:

$$\frac{x'^2 + y'^2}{R_{x'y'}} + \frac{z'^2}{R_{z'}} = 1 \tag{2.3}$$

In the final fitting phase, the radii and angulations are determined while the center is fixed to the coordinates calculated in the second fitting phase. The duration of the fitting process depends on the size of the evaluated retinal fraction, with a duration of half a minute or less for the central retina and three to four minutes for the more peripheral retina.

3. 2D FITTING ALGORITHM

For the 2D-fits, equation 2.1 was adjusted to define an ellipse instead of ellipsoid:

$$\frac{x'^2}{R_{x'}} + \frac{z'^2}{R_{z'}} = 1 \tag{3.1}$$

were $R_{x'}$ and $R_{z'}$ are the radii of the ellipse along the principal axes x' and z' of the used slice. As such, principal axis x' is oriented in the left-right direction for the transversal slices and in the feet-head direction for sagittal slices. Furthermore, the described fitting protocol was adjusted in all four steps to ignore the 2nd dimension. To that end, parameters $C_{y'}$, β and y' of steps 1, 2 and 3 were disregarded, and equation 2.2 was redefined as:

$$d^2 = (R_{x'} \cos\theta - x')^2 + (R_{z'} \sin\theta - z')^2 \tag{3.2}$$

REFERENCES

1. Kraft D. A software package for sequential quadratic programming. *Forschungsbericht-Deutsche Forschungs- und Versuchsanstalt fur Luft- und Raumfahrt* 1988

2. Quasi-Newton Methods. Numerical Optimization. New York, NY: Springer New York 2006:135-63.