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Time to replace the spherical equivalent with the average paraxial lens power

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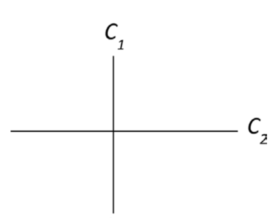
A scalar measure of refractive power is frequently used to evaluated refractive outcomes. Such scalar measures are based on the average power of a lens or lens system, for example, spherocylinder.

A commonly used measure has been the mean spherical equivalent usually referred to as the spherical equivalent (SE). Harris recognised, however, that there may be other scalar terms for the SE.[1] As such, it has recently been shown that the A_pP, which in addition to orthogonal rays, includes the average or mean of oblique paraxial rays. It is also associated with better visual acuity than the SE and therefore may be of more clinical use.[2] The relationship between the A_pP and SE, however, needs clarification.

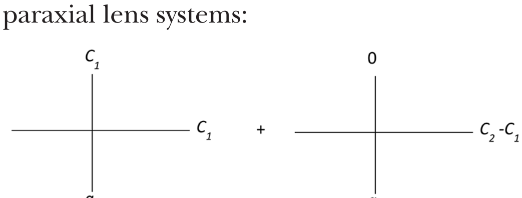
There are currently two methods used to calculate the average paraxial power of a lens:

- SE, which is derived from the average orthogonal paraxial power[3] $\bar{F}_{SE(orthogonal)}$.
- Average paraxial power (A_pP), which is derived from the average orthogonal and oblique paraxial powers[2] $\bar{F}_{A_pP(orthogonal+oblique)}$.

Consider the following lens system F written as an optical cross where C_1 and C_2 are two orthogonal lens cylinders at axes a and a±90:



This can be separated into the sum of two paraxial lens systems:



often then written in sphere/cylinder x-axis ($S/C_x a$) form as $S_1/(C_2 - C_1)_x a$, based on the assumption that two equal orthogonal cylinders equal a spherical lens, that is, $C_{1x}a + C_{1x}a_{\pm 90} = S_1$.

The average power $\bar{F}$ of the system is the sum of the average power of each component, that is, $\bar{F} = \overline{C_{1xa}} + \overline{C_{1xa\pm90}} + \overline{(C_2 - C_1)_x a}$ where $\overline{C_{1xa}}$, $\overline{C_{1xa\pm90}}$ and $\overline{(C_2 - C_1)_x a}$ are the average paraxial powers of each of the three lenses.

For the SE, the average is the average of orthogonal paraxial rays, that is,

SE=average orthogonal paraxial power $\bar{F}_{SE(orthogonal)}$

$$\bar{F}_{SE(orthogonal)} = \frac{C_1}{2} + \frac{C_1}{2} + \frac{C_2 - C_1}{2},$$

So that

$$SE = \bar{F}_{SE(orthogonal)} = \frac{C_1}{2} + \frac{C_2}{2}.$$

While for the A_pP, the average is the average of orthogonal and oblique paraxial rays, that is,

A_pP=average orthogonal and oblique paraxial power $\bar{F}_{A_pP(orthogonal+oblique)}$,

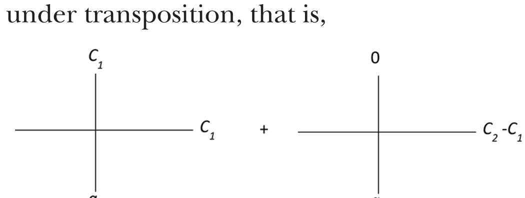
$$\bar{F}_{A_pP(orthogonal+oblique)} = \frac{C_1}{4} + \frac{C_1}{4} + \frac{C_2 - C_1}{4},$$

$$A_pP = \bar{F}_{A_pP(orthogonal+oblique)} = \frac{C_1}{4} + \frac{C_2}{4}.$$

Therefore, the A_pP is equal to half the SE if calculated in cross cylinder form, that is, $A_pP = \frac{SE}{2}$.

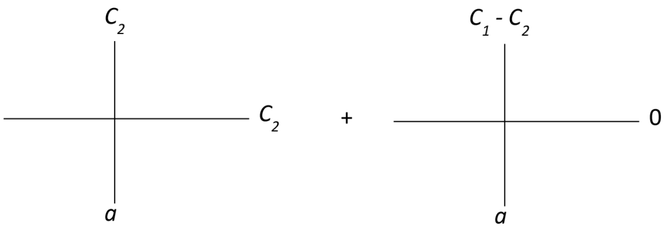
TRANSPOSITION AND A_pP

Analogous to the SE, the A_pP also holds under transposition, that is,



$$A_pP = \frac{C_1}{4} + \frac{C_1}{4} + \frac{(C_2 - C_1)}{4} + \frac{0}{4} = \frac{C_1}{4} + \frac{C_2}{4},$$

and after transposition



$$A_pP = \frac{C_2}{4} + \frac{C_2}{4} + \frac{(C_1 - C_2)}{4} + \frac{0}{4} = \frac{C_2}{4} + \frac{C_1}{4}.$$

TRANSFORMATION

Transforming an optical cross of cylinders into a spherocylinder $S/C_x a$ often rests on the assumption that two equal orthogonal cylinders equal a spherical lens,[3] that is, $C_{1x}a + C_{1x}a_{\pm 90} = S_1$ or $C_{2x}a + C_{2x}a_{\pm 90} = S_2$. This is an approximation, as it has been shown that two equal orthogonal cylinders are not equal to a spherical lens[3] and in addition, the intersection of two equal orthogonal cylinders is Steinmetz solid rather than a sphere.[2] It would, therefore, be more accurate to state that within a paraxial ray system that two equal orthogonal cylinders only approximate a spherical lens.

If $C_{1x}a + C_{1x}a_{\pm 90} \cong C_{1S}$, where C_{1S} represents the resultant spherical lens, S_1; and $C_{2x}a + C_{2x}a_{\pm 90} \cong C_{2S}$ where C_{2S}, represents the resultant spherical lens S_2.

Then, based on this approximation, the lens system is written in $S/C_x a$ notation as:

$$S_1/(C_2 - C_1)_x a \text{ or after transposition } (\leftrightarrow)$$

$$S_1/\left(C_2 - C_1\right)_x a \leftrightarrow S_1 + \left(C_{2S} - C_{1S}\right) / -\left(C_2 - C_1\right) = S_2/\left(C_1 - C_2\right)_x a_{\pm 90}$$

$$S_2/(C_1 - C_2)_x a \pm 90$$

It is important to note, however, that the derivation of the SE is based on treating a lens cylinder as a cylinder and not on the assumption that two orthogonal cylinders equate to a spherical lens.

Although not strictly correct, the SE is often calculated as:

$$SE \cong S_1 + \frac{(C_2 - C_1)}{2} \text{ or after transposition}$$

$$SE \cong S_2 + \frac{(C_1 - C_2)}{2}$$

where $\frac{(C_2 - C_1)}{2} = \overline{C_2 - C_1}$ and $\frac{(C_1 - C_2)}{2} = \overline{C_1 - C_2}$

It is also important to note that the error introduced by this approximation is not constant and the magnitude of the error depends on the power of the two lenses.[3] Therefore, application of the formula for the SE based on the assumption that two orthogonal lenses equate to a spherical lens should be limited to small powers.

If a refractive error, however, does comprise a true spherical component, then the average of that component is of course the sphere.

Therefore, for a refractive power that contains a sphere, the $A_pP = Sphere + \frac{C}{4}$ and the $SE = Sphere + \frac{C}{2}$

and the difference between the A_pP and SE is equal to $\frac{C}{4}$.

CONCLUSION

While any scalar measure of refractive error loses dimensions of sensitivity, scalar measures of power remain important in many applications. The A_pP as a scalar measure is more inclusive and appears to be associated with better visual acuity than the SE, although further clinical trials are needed particularly in different age groups and conditions. It will also be important to explore and evaluate the A_pP with higher refractive errors. If the evidence remains supportive, then useful clinical applications of the A_pP would include, for example, providing an equivalent lens for a person who is unable to tolerate the toric prescription as this is associated with less degradation of visual acuity than may occur with the SE.

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