Editorial

Time to replace the spherical equivalent with the average paraxial lens power

Stephen B Kaye

A scalar measure of refractive power is frequently used to evaluate refractive outcomes. Such scalar measures are based on the average power of a lens or lens system, for example, sphero-cylinder.

A commonly used measure has been the mean spherical equivalent usually referred to as the spherical equivalent (SE). Harris recognised, however, that there may be other scalar terms for the SE. As such, it has recently been shown that the $A_p^p$, which in addition to orthogonal rays, includes the average or mean of oblique paraxial rays. It is also associated with better visual acuity than the SE and therefore may be of more clinical use. The relationship between the $A_p^p$ and SE, however, needs clarification.

There are currently two methods used to calculate the average paraxial power of a lens:

- SE, which is derived from the average orthogonal paraxial power $F_{SE(orthogonal)}$.
- Average paraxial power ($A_p^p$), which is derived from the average orthogonal and oblique paraxial powers $F_{ApP(orthogonal+oblique)}$.

Consider the following lens system $F$ written as an optical cross where $C_1$ and $C_2$ are two orthogonal lens cylinders at axes $a$ and $a\pm90$:

This can be separated into the sum of two paraxial lens systems:

Often then written in sphere/cylinder form as $S_1/(C_2-C_1)x^a$, based on the assumption that two equal orthogonal cylinders equal a spherical lens, that is, $C_1a + C_2a_{\pm90} = S_1$.

The average power $F$ of the system is the sum of the average power of each component, that is, $F = \bar{F}_{1x^a} + \bar{F}_{1x^\pm90} + (\bar{C}_2 - \bar{C}_1)x^a$

where $\bar{C}_1x^a$, $\bar{C}_1x^\pm90$ and $(\bar{C}_2 - \bar{C}_1)x^a$ are the average paraxial powers of each of the three lenses.

For the SE, the average is the average of orthogonal paraxial rays, that is,

$SE = \bar{F}_{SE(orthogonal)}$ = $\frac{C_1}{2} + \frac{C_2}{2} + \frac{C_2-C_1}{2}$,

So that

$SE = \bar{F}_{SE(orthogonal)} = \frac{C_1}{2} + \frac{C_2}{2}$.

While for the $A_p^p$, the average is the average of orthogonal and oblique paraxial rays, that is,

$A_p^p = \bar{F}_{ApP(orthogonal+oblique)}$,

$A_p^p = \bar{F}_{ApP(orthogonal+oblique)} = \frac{C_1}{2} + \frac{C_2}{2} + \frac{C_2-C_1}{2}$,

$A_p^p = \bar{F}_{ApP(orthogonal+oblique)} = \frac{C_1}{2} + \frac{C_2}{2}$.

Therefore, the $A_p^p$ is equal to half the SE if calculated in cross cylinder form, that is, $A_p^p = \frac{SE}{2}$.

TRANSPOSITION AND $A_p^p$

Analogous to the SE, $A_p^p$ also holds under transposition, that is,
\[ ApP = \frac{C_1}{4} + \frac{C_2}{4} + \frac{(C_2 - C_1)}{4} + \frac{9}{4} = \frac{C_1}{4} + \frac{C_2}{4}, \]

and after transposition

\[ C_1 - C_2 \]

\[ C_2 - C_1 \]

\[ C_2 \]

\[ 0 \]

\[ \frac{9}{4} \]

\[ \frac{C_1}{4} + \frac{C_2}{4} \]

TRANSFORMATION

Transforming an optical cross of cylinders into a spherocylinder \( S/C_4 \) often rests on the assumption that two equal orthogonal cylinders equal a spherical lens,\(^3\) that is, \( C_1a + C_1a_{\pm 90} = S_1 \) or \( C_2a + C_2a_{\pm 90} = S_2 \). This is an approximation, as it has been shown that two equal orthogonal cylinders are not equal to a spherical lens\(^1\) and in addition, the intersection of two equal orthogonal cylinders is Steinmetz solid rather than a sphere.\(^2\) It would, therefore, be more accurate to state that within a paraxial ray system that two equal orthogonal cylinders only approximate a spherical lens.

If \( C_1a + C_1a_{\pm 90} \equiv C_1S \), where \( C_1S \) represents the resultant spherical lens, \( S_1 \); and \( C_2a + C_2a_{\pm 90} \equiv C_2S \) where \( C_2S \), represents the resultant spherical lens \( S_2 \).

Then, based on this approximation, the lens system is written in \( S/C_4 \) notation as:

\[ S_1 / (C_2 - C_1) a \text{ or after transposition } (\leftrightarrow) \]

\[ S_1 / (C_2 - C_1) a \leftrightarrow S_1 + (C_2 - C_1) / - (C_2 - C_1) = S_2 / (C_1 - C_2), a_{\pm 90} \]

\[ S_2 / (C_1 - C_2) a \pm 90 \]

It is important to note, however, that the derivation of the SE is based on treating a lens cylinder as a cylinder and not on the assumption that two orthogonal cylinders equate to a spherical lens.

Although not strictly correct, the SE is often calculated as:

\[ SE \cong S_1 + \frac{(C_2 - C_1)}{2} \text{ or after transposition} \]

\[ SE \cong S_2 + \frac{(C_1 - C_2)}{2} \]

where \( \frac{(C_2 - C_1)}{2} = C_2 - C_1 \) and \( \frac{(C_1 - C_2)}{2} = C_1 - C_2 \).

It is also important to note that the error introduced by this approximation is not constant and the magnitude of the error depends on the power of the lenses.\(^3\) Therefore, application of the formula for the SE based on the assumption that two orthogonal lenses equate to a spherical lens should be limited to small powers.

If a refractive error, however, does comprise a true spherical component, then the average of that component is of course the sphere.

Therefore, for a refractive power that contains a sphere, the \( ApP = \frac{C_1}{4} + \frac{C_2}{4} \) and the \( SE = \frac{C_1}{4} + \frac{C_2}{4} \)

and the difference between the \( ApP \) and SE is equal to \( \frac{C_1}{4} \).

CONCLUSION

While any scalar measure of refractive error loses dimensions of sensitivity, scalar measures of power remain important in many applications. The \( ApP \) as a scalar measure is more inclusive and appears to be associated with better visual acuity than the SE, although further clinical trials are needed particularly in different age groups and conditions. It will also be important to explore and evaluate the \( ApP \) with higher refractive errors. If the evidence remains supportive, then useful clinical applications of the \( ApP \) would include, for example, providing an equivalent lens for a person who is unable to tolerate the toric prescription as this is associated with less degradation of visual acuity than may occur with the SE.

Contributors SBK developed and wrote this editorial.

Funding The authors have not declared a specific grant for this research from any funding agency in the public, commercial or not-for-profit sectors.

Competing interests None declared.

Patient consent for publication Not required.

Ethics approval Not applicable.

Provenance and peer review Commissioned; externally peer reviewed.

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